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# DIAGRAMS, CYCLIC ORDERINGS AND ARISTOXENIAN SYNTHESIS

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*Introduction*

The broad polarisation of classical Greek music theory into the Pythagorean and Aristoxenian traditions reflects the tension between their respective numerological and phenomenological approaches. Aristoxenus places his concept of synthesis at the heart of melodic organisation. He identifies the Harmonicists as his predecessors, and is critical of their work with regards to synthesis, most notably the *katapyknōsis* (compression) of their diagrams of cyclically ordered scales. Claudius Ptolemy also attempts to reconcile reason and perception, taking a divisional top-down approach to melodic organisation. He catalogues tetrachord species, both his own and those of other theorists. Both Aristoxenus and Ptolemy object to the use of the *aulos* as a theoretical basis. More generally it is difficult to classify the 'instrumentalists' as theorists or performers and, if the former, position them on a theoretical spectrum of reason and perception.

This paper employs contemporary multi-dimensional diagrammatic representations of the various tetrachord species, as catalogued by Ptolemy. The representational considerations in these diagrams highlight the significance of function as distinct from interval magnitude and note position. Extending the pitch range of these diagrams and combining the genera provides visual representation of tetrachords in a wider context, illustrating, for example, the role of epimoric ratios in the scale species of Archytas. A multidimensional consideration of *katapyknōsis* as 'compression' and 'close-packing' provides an organisational framework that reconciles Aristoxenus' references to 28 *dieses* and the octave, and does so in a manner that is consistent with the Harmoncists' approach and Aristoxenus' criticisms of them. The theories of Schlesinger, based on the aulos, may be considered as an arihmetic extreme and counterpart to a purely geometric approach. This defines an arithmetic–geometric spectrum on which different theoretical approaches may be positioned, providing clarification on issues such as Ptolemaïs' problematic characterisation of the Pythagoreans.

*1 Background*

Despite difficulties surrounding the authenticity of writings on classical Greek music theory they have nonetheless been highly influential in subsequent theoretical developments [1, pp. 112-4]. Approaches are broadly polarised into the Pythagorean and Aristoxenian traditions, reflecting the tension between their respective numerological and phenomenological approaches.

The background material for the present study is divided into three subsections. The first introduces Aristoxenus' notion of synthesis as his basis of melodic organisation. The harmonicist predecessors of Aristoxenus, in particular their cyclic orderings and diagrams and his criticisms of them, are also presented. The second sub-section describes relevant aspects of Ptolemy's writings, in particular his theoretical approach and his catalogue of tetrachord species. The respective views of Aristoxenus and Ptolemy of the aulos as a scientificinstrument are deferred to the third sub-section, which also briefly introduces the instrumentalists (*organikoi*) and the *aulos*-centred work of Schlesinger.

*1.1 Aristoxenus*

*1.1.1 Synthesis*

Aristoxenus places his concept of synthesis at the heart of melodic organisation. It is based on the combination of intervals such that they form systems (*systēmata*) and Aristoxenus seeks 'some definite principle governing the synthesis of every interval with every other' [4, p. 129]. The principles by which such systems are constructed are closely connected to the notion of melodic continuity and succession, while the relations between such systems are associated with modulation.

*Melodic Continuity and Succession.*

Aristoxenus defines incomposite intervals, which are the fundamental units of synthesis, in terms of succession [4, p. 172], that is to say 'an incomposite interval is one bounded by successive notes'. The term 'succession' (*to hexēs*) is largely used interchangeably with 'continuity' (*synecheia*) [4, p. 129][1]. A similar third term 'consecution (*agōgē*) is applied to melodies, while the first two terms tend to relate to the structure of the underlying systems in which these melodies exist [4, p. 147]. The 'nature of melody' remains central to Aristoxenus' notion of continuity and succession [4, p. 166].

Aristoxenus clearly distinguishes an interval's magnitude from its function, with each depending on hearing and reason respectively [4, p. 150]. He stipulates in particular that magnitude gives no indication of function or the distinction between composite and incomposite intervals [4, p. 156] upon which the systems of melodic succession are based.

*Modulation.*

Aristoxenus explicitly relates the 'study of modulation' to the 'theory of melody', describing modulation in terms of 'the mutual affinities of *systēmata*' and 'the melody that occurs between one *systēma* and another' [4, p. 131]. He distinguishes melodies as simple or modulating, and gives indications of a systematic treatment but unfortunately his more detailed accounts of this subject have not survived [4, p. 154]. However, he does specifically point out the significance of the position (*topos*) for each *systēma* and refers to the *tonoi* in which each *systēma* lies 'when the melody can move from one to the other' [4, p. 131]. This supports the assertion that 'the intervallic relations between the fixed notes of one *systēma* and another were important' to Aristoxenus in the context of modulation [3, p. 189].

*1.1.2 Harmonicists*

Aristoxenus acknowledges his predecessors in the empirical study of harmonic science, calling them 'harmonicists' (αρμονικοί) [4, p. 124]. This acknowledgement is rarely in a flattering light; nonetheless, his mentioning them constitutesconsiderable recognition in comparison with the Pythagoreans who are barely mentioned at all [2, pp. 1-2]. The harmonicists are a broad category of theorists rather than a specific school [2, pp. 2-7].

Aspects of the harmonicists' work that are of interest here are their cyclic orderings, their diagrams and their association with the *aulos*. Of particular relevance is Aristoxenus' criticism of these aspects of their work with respect to synthesis (described above) as well as the wider study of harmonic science.

*Cyclic Orderings.*

Eratocles is a theorist known to us through Aristoxenus [4, p. 124], most likely active in the late 5th century B.C. [6, p. 50]. Despite his systematic approach not being typical of harmonicists in general [6, pp. 43,55], Aristoxenus treats him as a representative if somewhat more insightful example

of them [4, pp. 124,129], [2, p. 7]. While Eratocles' results appear to have had some correspondence with the musical practices of the time [6, p. 44], this is not necessarily the case for some aspects of his theories [6, p. 51]. Eratocles is primarily identified for his cyclic orderings of the intervals of the enharmonic octave [3, p. 187], that is the rotation of the steps of the enharmonic scale to derive a set of seven octave species. These cyclic orderings would appear to have been displayed in the diagrams of the type described below [4, p. 132], [6, p1. 53]. He is also identified as recognising the alternative melodic sequences arising on each side of the interval of a fourth (most likely referring to the possible conjunction and disjunction of tetrachords) [4, p. 129].

Aristoxenus criticises Eratocles' work primarily on the basis that it is incomplete, and also that it is 'without demonstration' (*i.e.* it fails to specify the principles on which the results are founded) [6, p. 153]. The scheme of cyclic orderings is considered incomplete with respect to genus (since only the enharmonic is considered), and also with respect to magnitude in that only octave species are given: in particular, despite his apparent awareness of the structural significance of the fourth [6, p. 54], Eratocles neglects to enumerate the possible species of fourth and fifth, or to demonstrate any principles by which species of fourth and fifth may be included or excluded from his scheme. In this regard he also fails to elaborate on the principles of melodic sequence surrounding the fourth, or indeed the systematic combination of intervals more generally [4, p. 129].

Aristoxenus does not necessarily dispute Eratocles' assertion of seven enharmonic octave species [6, p. 194-195]. Nor indeed does he appear to object to the mechanism of cyclic ordering, which is present in his definition of the forms (*eidē*) of the fourth [4, p. 184]. However, whilst his criticisms are largely procedural [6, p. 223], he also considers Eratocles' results false in that they do not correspond to the authority of perception [6, p. 153]. Despite these complaints, aspects of Eratocles' work may well have provided significant inspiration for Aristoxenus' theories [2, pp. 17-18].

*Diagrams.*

Aristoxenus associates the harmonicists with diagrams on several occasions [4, p. 125], [2, pp. 5-6]. Whilst the exact nature of their diagrams is not certain [1, p. 118], they would appear to have shared with Aristoxenus a geometric rather than arithmetic conception of pitch [4, p. 124], representing intervals on a line of evenly spaced quarter-tones onto which they mapped scales [2, p. 8], in particular the enharmonic octave species such as those of Eratocles' cyclic orderings [6, p. 153]. The intended purpose of these diagrams appears to be the comparison of scale structures [4, p. 125], [2, pp. 8-9].

The term *katapyknōsis* (καταπύκνωσις) is specifically associated with the harmonicists' diagrams [2, p. 6], and may be interpreted as 'compression' into a compact range of pitch and/or 'close-packing' of intervals within this range [3, p. 188]. This term is central to the two principal objections expressed by Aristoxenus' with regard to the harmonicists' diagrams.

The first objection relates to melodic continuity [3, p. 188], [2, p. 9]. Aristoxenus' contention is specifically that the succession of quarter-tones displayed in the harmonicists' compressed diagrams has no melodic validity [4, p. 145]. He explicitly identifies *katapyknōsis* as treating 'the proper course (*agōgē*) of melody with contempt' [4, p. 166] and clearly states that '*katapyknōsis* is unmelodic and altogether useless' [4, p. 154].

The second objection relates to modulation between *systēmata* [4, pp. 131-132], [3, pp. 188-189], [2, p. 8], whereby the harmonicists' diagrams are deemed to misrepresent pitch relations between scales by disregarding their respective positions [2, p. 16]. While this criticism again refutes the validity of *katapyknōsis*, the harmonicists are said to have touched on aspects of modulation by chance [4, p. 131]. In this respect they do not eliminate the possibility of modulation but rather stumble upon aspects of it unintentionally and without addressing the topic satisfactorily [3, pp. 190,192]. There is a suggestion that the harmonicists failed to exploit a potential application of their diagrams [2, p. 8].

The overall context in which the term *katapyknōsis* is used suggests it is a process whereby an uncompressed diagram is transformed into its compressed counterpart [3, pp. 194-195]. Furthermore, Aristoxenus' objections are directed specifically towards *katapyknōsis* rather than diagrammatic representation in general [2, pp. 6-7], which would seem to suggest that either Aristoxenus did not have access to an uncompressed form of the diagrams, or if he did he chose to refer only to the final compressed form[2].

*1.2 Ptolemy*

The *Harmonics* of Claudius Ptolemy, believed to have been written in the latter part of his scientific career in the second century A.D., is a significant if somewhat neglected treatise on music [4, p. 270]. Aspects of Ptolemy's work considered here are his theoretical approach, particularly in relation to that of Aristoxenus, and his catalogue of tetrachord species.

### *1.2.1 Theoretical Approach*

Ptolemy distinguishes himself from both the Pythagoreans and Aristoxenians on the grounds that they fail to find the correct balance between reason and perception [4, p. 279]. However, while Ptolemy recognises the importance of perception in validating reason, he retains mathematical reasoning as his basis of the principles of harmonic science, and along with his arithmetic rather than geometric quantification of pitch, and the cosmological aspects of his work, Ptolemy clearly defines himself in Pythagorean rather than Aristoxenian terms [4, pp. 270-271].

For his classification of notes Ptolemy adopts a hierarchy of homophones, concords and melodics, connected by a continuum of epimoric ratios (*i.e.* 2:1, 3:2, 4:3, 5:4, 6:5, 7:6, . . . ) [4, pp. 289-290]. Such epimoric ratios are a prerequisite for melodically successive intervals in Ptolemy's scheme [4, p. 286] which, in particular, excludes the possibility of any interval being divided exactly in half, a feature that is central to the Aristoxenian approach [4, p. 304].

In contrast to Aristoxenian synthesis, whereby composite intervals are assembled from incomposite intervals in a bottom-up manner, Ptolemy adopts a divisional approach working top-down through his hierarchy [4, pp. 307-309] and he explicitly objects to the bottom-up derivation of intervals [4, p. 337].

Ptolemy is critical of the Aristoxenians for comparing only the intervals between the notes rather than the notes themselves [4, p. 293]: This is also apparent in Ptolemy's distinction between function and position (*thesis*) [4, p. 325]), as opposed to the function and *magnitude* of Aristoxenus[3].

### *1.2.2 Tetrachord Species*

Of particular interest to the present study is Ptolemy's account of tetrachord species. As well as his own variants [4, pp. 309-314], Ptolemy also examines the species of other theorists, namely Aristoxenus [4, p. 303], Archytas [4, p. 304] and Didymus [4, pp. 342-349], with additional references to those of Eratosthenes [4, pp. 346-349].

Tables 1, 2 and 3 describe the tetrachord species catalogued by Ptolemy for each of the three *genera*: the diatonic, chromatic and enharmonic respectively[4]. In each case absolute frequency ratios are specified in descending pitch from a perfect fourth (4:3) to the tonic (1:1), interspersed with the ratios representing the magnitudes of intervals between neighbouring notes.[5]

## *1.3 Auloi*

### *1.3.1 The aulos as a Scientific Instrument*

Aristoxenus associates at least some of the harmonicists with the aulos as their basis for the study of harmonic science, both as a means of modal organisation [4, pp. 153-154] and harmonic attunement [4, pp. 156-158]. While Aristoxenus is content to refer to the aulos in illustrating the overall pitch range in practical use [4, p. 139], he finds the harmonicists' theoretical applications of the *aulos* specifically, and of instruments in general, highly objectionable.

Ptolemy is also critical of the suitability of the *aulos* for demonstrating pitch relations, and many of his practical objections echo those of Aristoxenus. However, Ptolemy's remarks are made in conjunction with asserting the suitability of the string of the kanon or monochord as a scientific instrument, not as the basis for the deduction of principles but rather to demonstrate the agreement of the principles with perception [4, p. 291].

### *1.3.2 The Instrumentalists (organikoi)*

As well as the specific association of the harmonicists with the aulos, Aristoxenus also acknowledges the perceptual approach of 'those who worked on instruments' with some ambiguity as to whether they were performers or theorists [4, p. 152].

The 'instrumentalists' (*organikoi*) of Ptolemaïs and Didymus [4, pp. 241-242] are placed at the perceptual end of a theoretical spectrum. They are dissociated from reason and its representatives the Pythagoreans [6, pp. 438-439], but nonetheless they would seem to be considered theorists in the sense that the spectrum they are placed on is defined as a theoretical one. A problematic aspect is Ptolemaïs characterisation of the Pythagoreans as giving equal status to reason and perception [6, p. 440].

A key distinction for Aristoxenus is that between 'the audible notes produced' from the aulos and the construction of 'the instrument itself', that is to say the quantification of pitch in the auditory (geometric) rather than physical (arithmetic) domain [6, p. 58].

*1.3.3 Kathleen Schlesinger's 'The Greek Aulos'*

Kathleen Schlesinger, the twentieth-century musicologist, could certainly be characterised as an instrumentalist. Her theories [8] are based extensively on the physical properties of auloi, with her scales defined using a strictly arithmetic quantification of pitch. This places her work in stark contradiction to that of Aristoxenus [4, pp. 154-155].

Whilst the applicability of Schlesinger's scales to any period in the history of Greek music is highly questionable, her work is of interest in a speculative capacity [9, pp139,143].

*2 Tetrachord Diagrams*

Tetrachords such as those identified in Section 1.2.2 are traditionally depicted in tabular form. Where they are represented diagrammatically this tends to be in a single dimension (i.e. on a line) where distances are typically measured arithmetically (corresponding to lengths on a monochord string) or, less frequently, geometrically (as in the diagrams of the Harmonicists). The diagrams in this section employ a multidimensional representation of pitch space which integrates arithmetic and geometric elements of pitch such that any given interval may be represented by a vector having the same magnitudeand direction regardless of its location [7]. Figures 1, 2 and 3 represent the diatonic, chromatic and enharmonic genera of the tetrachord species corresponding to the data in Tables 1, 2 and 3 respectively. Each tetrachord instance is represented by a path from the fixed tonic (1:1) to the perfect fourth (4:3), ascending in pitch from left to right and passing through two intermediate notes. Each note's position is determined in relation to a central reference axis of 10 quarter-tones[6], the note being displaced vertically in proportion to its difference from its nearest quarter-tone reference point.

*3 Observations and Analysis*

*3.1 Function*

In figures 1 to 3, most of the ratios describing notes stand unambiguously in relation to their nearest quarter-tone reference point. However, some ratios require more detailed consideration in their placement.

*The Ratio 28:27*

The note represented by the ratio 28:27 is used in several tetrachord species. Its logarithmic value equates to 1.26 quarter-tones, that is to say its intervallic magnitude is much closer to a single quarter-tone (1 + 0.26) than it is to a semitone (2 − 0.74). In the enharmonic genus this note's functional role is consistent with its proximity to a single quarter-tone, and it is depicted as such. However, in the diatonic and chromatic genera, it is depicted as a very small semitone (*i.e.* relative to 2 quarter-tones) in order to reflect its functional role in these contexts[7].

*The Ratios 7:6 and 8:7*

The magnitudes of the ratios 7:6 and 8:7 are both closest to five quartertones (5.34 and 4.62 quarter-tones respectively), but are depicted in the diatonic and chromatic diagrams in relation to six and four quarter-tones respectively in accordance with their functional roles: 7:6 is the incomposite upper interval of Ptolemy's tense chromatic, as opposed to its role in his soft diatonic where it forms the lower composite interval divided as a semitone (21:20) and tone (10:9)[8].

*The Ratio 12:11*

In Ptolemy's even diatonic species, the note at 12:11 divides the interval 6:5 as 12:11 and 11:10. Here each of the two latter intervals are just sharper than three quarter-tones (3.01 and 3.30 quarter-tones respectively); indeed both are closer to a wholetone (4 quarter-tones) than they are to a semitone (2 quarter-tones). Figure 1 depicts the note at 12:11 such that it reflects the somewhat anomalous division of 6:5 as 3+3 quarter-tones rather than the more conventional 2+4 (semitone+tone) to which the other diatonic species conform. Conversely the functional role of the central 12:11 interval in Ptolemy's tense chromatic species is more appropriately depicted in figure 2 as a large semitone, preserving the chromatic function of the interval 8:7 as a large (and divisible) tone.

*3.2 Archytas' Scale Species*

Additional observations may be made from diagrams which combine tetrachords into larger structures. One such structure is the combination of the diatonic, chromatic and enharmonic tetrachord species of Archytas, arranged as a pair of disjunct tetrachords spanning an octave – an arrangement which raises several theoretical issues [4, 46-52]. Figure 4 depicts this structure in an exploded, twodimensional form in which the solid lines represent melodically successive scale steps while the dashed lines indicate additional structurally significant intervals.

A notable feature is the structural symmetry of intervals between the diatonic (4:3 = 32:27 + 9:8) and chromatic (4:3 = 9:8 + 32:27) tetrachords such that the *leimma* (256:243), whilst not occurring as a melodic step, is structurally significant to Archytas in the relation between these two genera [5, pp. 121-122].

Also apparent in figure 4 is the ratio of 6:5 between mese (4:3) and enharmonic paranete (8:5), and likewise 7:6 between mese and trite (14:9). Thisemployment of epimoric ratios in the theories of Archytas is something which Ptolemy does not seem to exhibit any awareness of [5, pp. 123-125].

*28 Dieses*

There is a problematic aspect of Aristoxenus' description of the harmonicists' diagrams in that he states that 'they have separated off just one magnitude, the octave… and devoted all their study to it' [4, p. 127] while later associating the diagrams with 28 consecutive dieses, where a diesis is taken to be an enharmonic diesis (*i.e.* quarter-tone) of which there are only 24 to the octave [4, p.145].

In attempting to reconcile 28 *dieses* to the octave, an ancient Dorian scale reported by Aristides Quintilianus and spanning an octave plus a tone (and therefore 28 *dieses*) provides a feasible basis of an explanation [3, pp. 188-192]. The present study proposes a configuration of Eratocles' cyclic octave species that would also appear to be consistent with all the surrounding facts.

Let the starting point be an uncompressed system of enharmonic octave species whose internal structures are cyclically related (as per Eratocles) but the notes of which retain their absolute positions in the system spanning two octaves, as shown in figure 5(a). Note that the sequence of scales is bounded by a pair of Dorian octave species (two tetrachords separated by a central disjunctive tone).

Note also that the sequence is centred on a pair of conjunct tetrachords with a tone above (Mixolydian) or below (Hypodorian). It is around this conjunction and pair of tones that a 28-diesis continuum may be constructed. The diagram may firstly be compressed in pitch range by relocating each scale such that its scale such that its lower and/or upper tetrachord coincides with that of its immediately neighbouring scales, as shown in figure 5(b). Whilst this compression is not strictly into the smallest possible pitch range (for octave species) of 24 dieses, it does fit logically into a range of 28 dieses. Furthermore the coincidence of tetrachords arising from this compression is consistent with Aristoxenus' assertions that the harmonicists disregarded the proper topos (*i.e.* absolute position) of scales while also accidentally stumbling on aspects of modulation between them.

The other aspect of katapyknōsis, that is the 'filling in' of intervals, is at least partly achieved by compressing the diagram vertically such that all scales are merged onto a single continuum, as in figure 5(c). Whilst this arrangement necessarily leaves some gaps, the diesis remains a theoretical unit, and the resulting misrepresentation of melodic succession is certainly consistent with Aristoxenus' criticism of the harmonicists' close-packing in this regard.

*3.4 Kathleen Schlesinger's 'The Greek Aulos'*

While there are examples of linear divisions in the above tetrachord species, most notably 12:11:10:9 in Ptolemy's even diatonic, the theory of Kathleen Schlesinger [8] represents a much more elaborate development of the concept of linear division.

Figure 6 presents an example of Schlesinger's modal system of the seven *harmoniai* in the Dorian *tonos* [9, 140]. From left to right, string lengths descend from 32 to 8 over an ascending pitch range from the tonic (1:1) to the double-octave (4:1). The double-octave system is centred on the Dorian octave species 22!11 whose upper tetrachord 14→11 is replicated in the lower octave as 28→22, and likewise the lower octave 22→16 is replicated in the upper octave as 11→8.

Note that the interval ratios broadly represent successively larger magnitudes as pitch ascends, and the only recurrence of particular interval magnitudes is the replication between the lower and upper octaves, and recurring composite intervals (some of which are indicated in the figure by dashed lines).

There are certainly incompatibilities between Schlesinger's ideas and those of Aristoxenus [4, p154], not least of which is the use of the aulos as a theoretical device. Nonetheless on a spectrum of purely arithmetic to purely geometric conception of pitch, Schlesinger's theory acts as a representative of the arithmetic extreme and, in a reductionist capacity, is perhaps a useful and complementary counterpart to purely geometric analyses such as Balzano(1980) [10]. Figure 7 depicts such a spectrum, with the purely geometric and arithmetic axes defining a space into which Aristoxenus, Ptolemy and other theoretical groups may be positioned.

This spectrum also suggests a possible explanation for Ptolemaïs' problematic characterisation of the Pythagoreans as giving equal status to perception and reason [6, p. 440]. In attempting to place the Pythagoreans on the arithmetic-geometric spectrum of figure 7 it is conceivable that the Pythagorean chromatic scale, with only two scale-step sizes (the tone and the *leimma*), is the arithmetically-based scale which is closest to a uniform geometric division with its single step size. In contrast, additional arithmetic elements (effectively going beyond the Pythagorean 1:2:3:4 to include ratios involving 5, 6, 7, etc.) introduce additional scale-step size distinctions, diverging further from a uniform division, with the process culminating in Schlesinger's arithmetic extreme where no two scale-step sizes are the same.

In this respect, the arithmetic-geometric spectrum overcomes ambiguities in the more traditional reason–perception spectrum: one may reason geometrically as well as arithmetically. It also clarifies the wider dichotomy between theory (often associated with reason) and practice, which is associated with both perception (geometric) and instruments (arithmetic).

## *4 Conclusion*

The representation of Ptolemy's catalogue of tetrachords in the form of twodimensional diagrams retains the numerical properties of the constituent notes and intervals while allowing the Aristoxenian notion of melodic continuity to be expressed in a manner which the Harmonicists' diagrams did not. Ptolemy's top-down divisional approach is represented just as effectively as the bottom-up synthesis approach of Aristoxenus.

In particular, representational considera-tions highlight the significance of function as opposed to interval magnitude or note position. While the precise functional roles of notes remain open to interpretation, the two-dimensional diagrams explicitly emphasise the issue in a visually intuitive manner. Exploded diagrams of larger structures allow tetrachords to be placed in context and wider structural characteristics and theoretical approaches to be understood, as in the case of Archytas' scale species and his use of epimoric ratios.

The exploration of the different aspects of the process of *katapyknōsis*, around a logical structural framework spanning 28 *dieses*, accommodates the 'compressed' and 'close-packed' diagrams of the Harmonicists', and does so in a manner consistent with Aristoxenus' descriptions and criticisms.

Kathleen Schlesinger's linear divisions based on the aulos, despite their questionable historical applicability, provide a useful theoretical counterpart to purely logarithmic treatments of pitch, each defining the respective arithmetic and geometric boundaries of a conceptual spectrum of approaches to pitch quantification. By positioning different theoretical approaches on this spectrum, insight can be gained into these approaches and the relationships between them, as with Ptolemaïs' characterisation of the Pythagoreans.

| Archytas | Eratosthenes | Didymus | Ptolemy | |
|---|---|---|---|---|
| | | | soft | tense |
| 4:3 | 4:3 | 4:3 | 4:3 | 4:3 |
| 32:27 | 6:5 | 6:5 | 6:5 | 7:6 |
| 9:8 | 10:9 | 10:9 | 10:9 | 8:7 |
| 243:224 | 19:18 | 25:24 | 15:14 | 12:11 |
| 28:27 | 20:19 | 16:15 | 28:27 | 22:21 |
| 28:27 | 20:19 | 16:15 | 28:27 | 22:21 |
| 1:1 | 1:1 | 1:1 | 1:1 | 1:1 |

Table 1: Diatonic tetrachord species from Ptolemy's Harmonics

| Archytas | Eratosthenes | Didymus | Ptolemy | |
|---|---|---|---|---|
| | | | soft | tense |
| 4:3 | 4:3 | 4:3 | 4:3 | 4:3 |
| 32:27 | 6:5 | 6:5 | 6:5 | 7:6 |
| 9:8 | 10:9 | 10:9 | 10:9 | 8:7 |
| 243:224 | 19:18 | 25:24 | 15:14 | 12:11 |
| 28:27 | 20:19 | 16:15 | 28:27 | 22:21 |
| 28:27 | 20:19 | 16:15 | 28:27 | 22:21 |
| 1:1 | 1:1 | 1:1 | 1:1 | 1:1 |

Table 2: Chromatic tetrachord species from Ptolemy's Harmonics

| Archytas | Eratosthenes | Didymus | Ptolemy |
|---|---|---|---|
| 4:3 | 4:3 | 4:3 | 4:3 |
| 5:4 | 19:15 | 5:4 | 5:4 |
| 16:15 | 20:19 | 16:15 | 16:15 |
| 36:35 | 39:38 | 31:30 | 24:23 |
| 28:27 | 40:39 | 32:31 | 46:45 |
| 28:27 | 40:39 | 32:31 | 46:45 |
| 1:1 | 1:1 | 1:1 | 1:1 |

Table 3: Enharmonic tetrachord species from Ptolemy's Harmonics

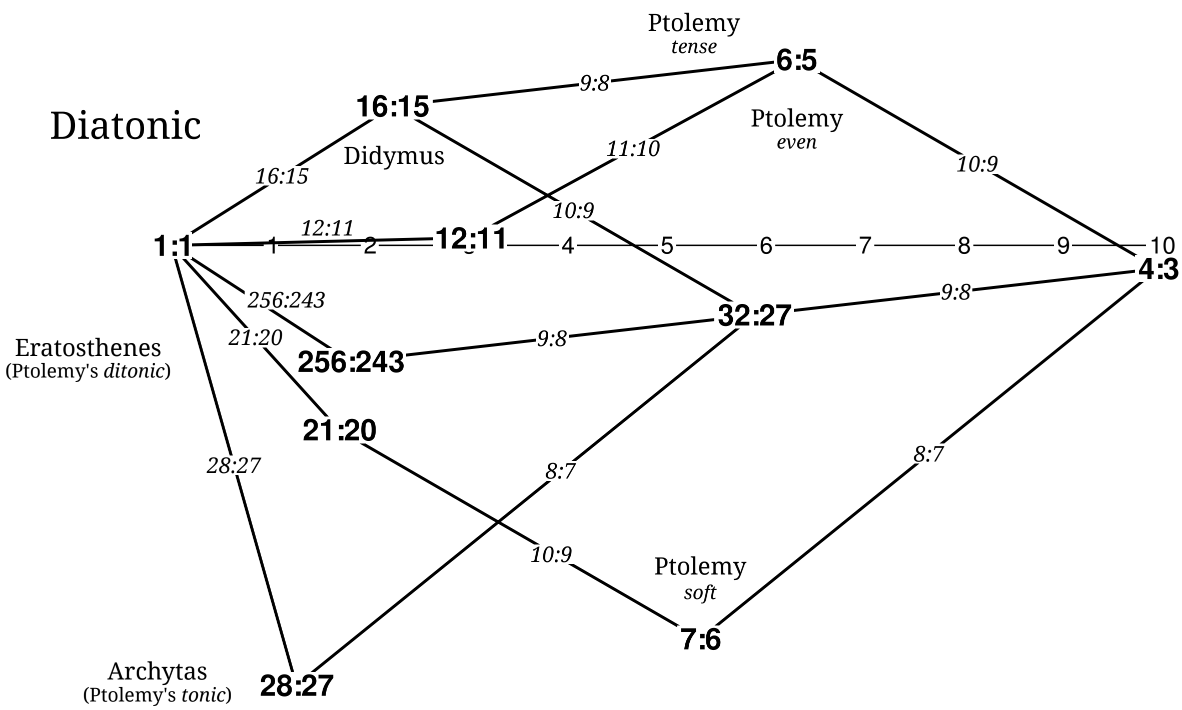


Fig. 1. Exploded diagram of the diatonic tetrachord species corresponding to Table 1.

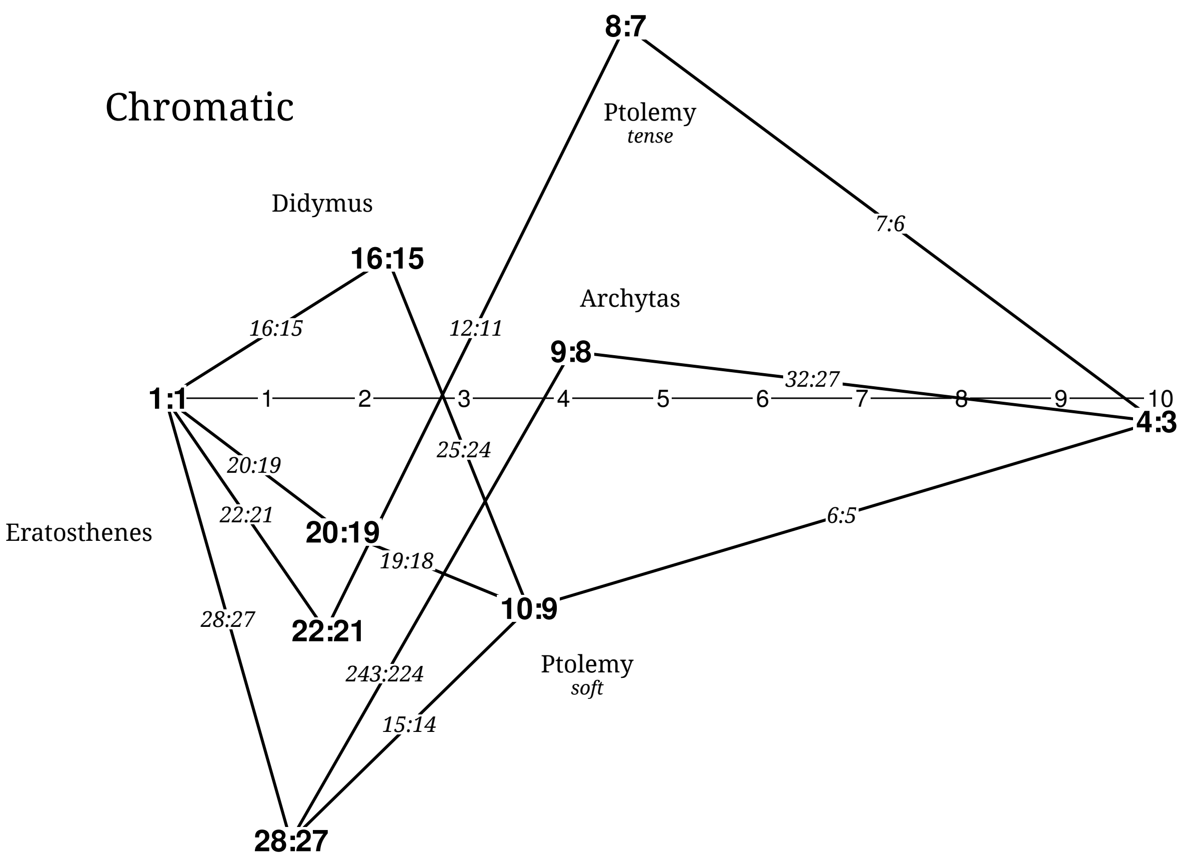


Fig. 2. Exploded diagram of the chromatic tetrachord species corresponding to Table 2.

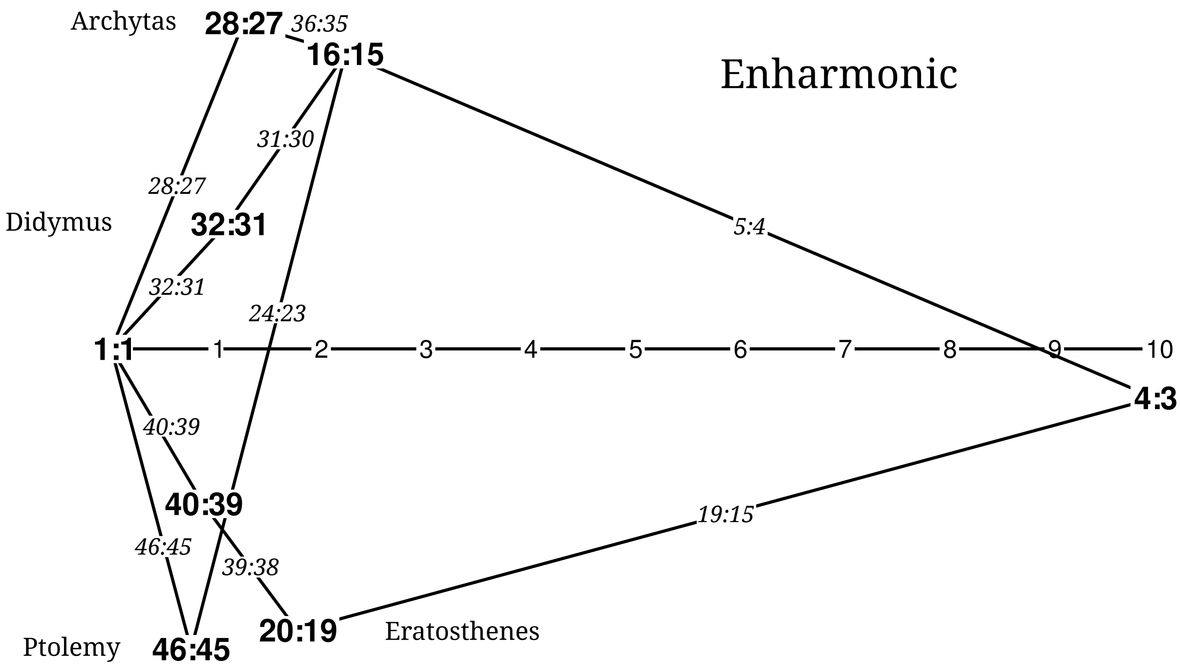


Fig. 3. Exploded diagram of the enharmonic tetrachord species corresponding to Table 3.

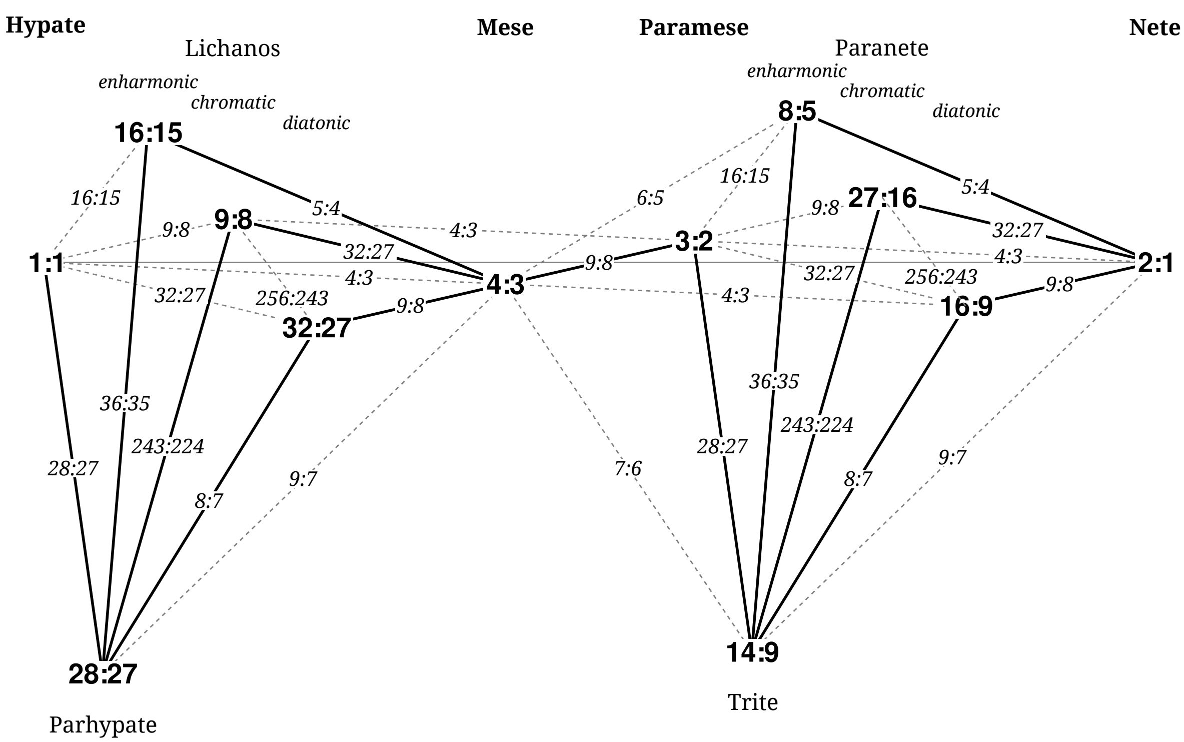


Fig. 4. Exploded diagram combining Archytas' diatonic, chromatic and enharmonic genera over the range of an octave.

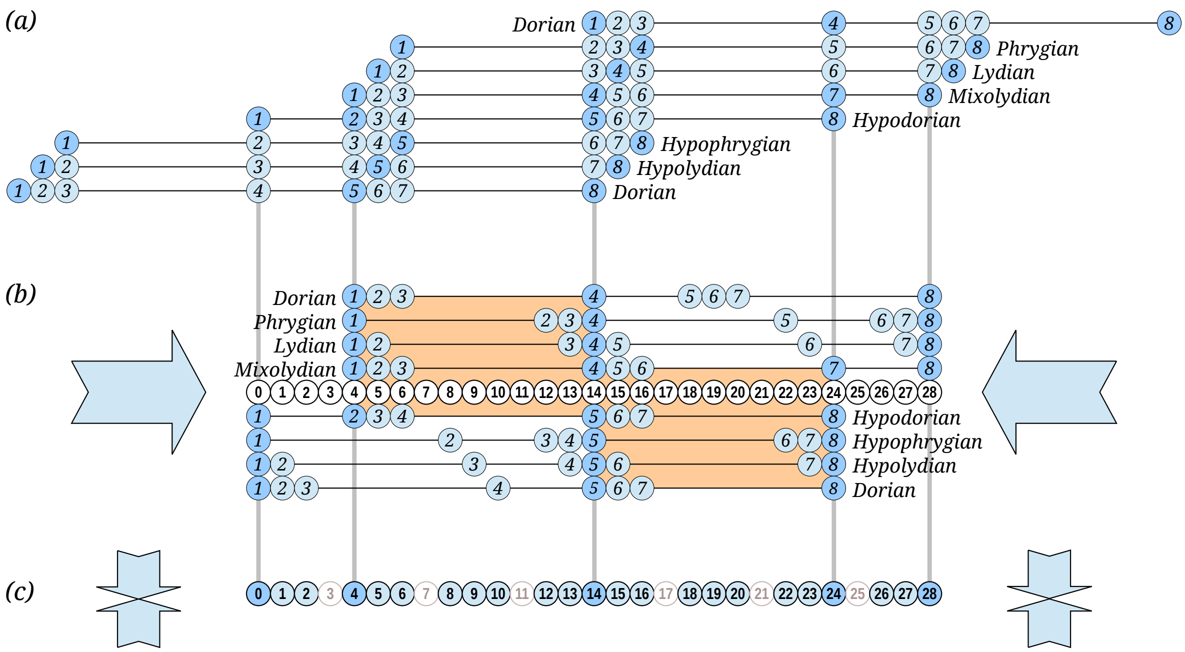


Fig. 5. (a) A hypothetical uncompressed diagram of cyclically related enharmonic octave species, subject to katapykn¯osis as (b) compression of pitch range and (c) filling-in of unit intervals over a 28-*diesis* range.

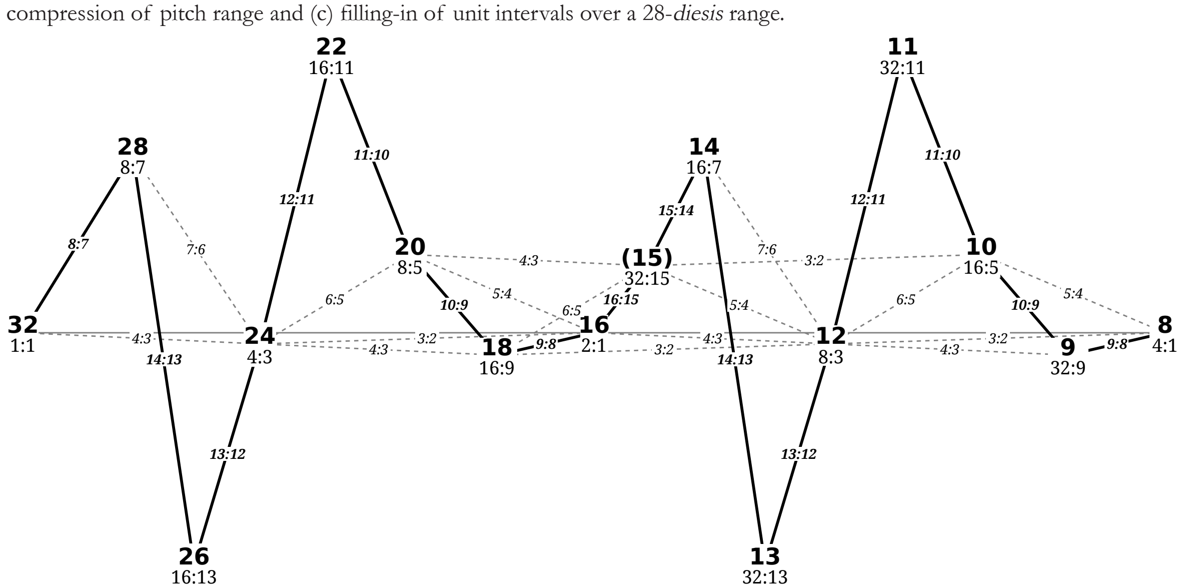


Fig. 6. An exploded diagram of Schlesinger's system of linear division, employing a descending sequence from 32 to 8 over an ascending pitch range from the tonic (1:1) to the double-octave (4:1). Each of the seven *harmoniai* are labelled at the lowest-pitched note of their octave range.

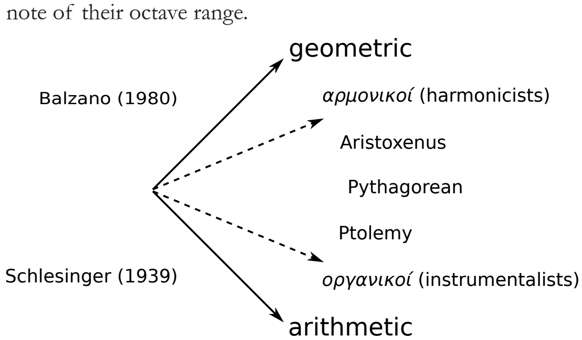


Fig. 7. (left) The spectrum of purely arithmetic and purely geometric conceptions of pitch, the respective extremes of which are represented by analyses such as Schlesinger(1939) and Balzano (1980).

## Notes

[1] The term succession is not only applied to incomposite intervals, but also to higher level components such as tetrachords [4, p. 172].

[2] The possibility of Aristoxenus being selective in his references is not unprecedented [6, p. 41, n. 12].

[3] This is not intended to imply that the Aristoxenian and Ptolemaic notions of function are equivalent, but simply to contrast 'position' and 'magnitude' in their respective approaches.

[4] The tetrachord species of Aristoxenus are uniquely described in geometric units rather than arithmetic ratios, and as they are not explicitly depicted in the subsequent diagrams they are not included here.

[5] Frequency ratios are used for modern convenience; Ptolemy himself quantified his tetrachord species as string lengths ascending as they descend in pitch.

[6] The unit obtained by dividing the octave (2:1) into 24 logarithmically equal parts.

[7] Compare the visual continuity of 46:45→40:39→32:31→28:27 in figure 3 (enharmonic) with that of 28:27→16:15 in figures 1 and 2 (diatonic and chromatic).

[8] Compare the visual continuity of 10:9→9:8→8:7 in figure 2 (chromatic) with 6:5→32:27→7:6 in figure 1 (diatonic).

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